

Scalar perturbation in warm tachyon inflation in LQC in light of Planck and BICEP2

M. R. Setare ^{a1}, V. Kamali ^{b2}

^a *Department of Science, Campus of Bijar, University of Kurdistan.
Bijar, Iran.*

^b *Department of Physics, Faculty of Science,
Bu-Ali Sina University, Hamedan, 65178, Iran*

Abstract

We study warm-tachyon inflationary universe model in the context of the effective field theory of loop quantum cosmology. In slow-roll approximation the primordial perturbation spectrums for this model are calculated. We also obtain the general expressions of the tensor-to-scalar ratio, scalar spectral index. We develop this model by using exponential potential, the characteristics of this model is calculated in great details. The parameters of the model are restricted by recent observational data from Planck, WMAP9 and BICEP2.

¹E-mail: rezakord@ipm.ir

²E-mail: vkamali1362@gmail.com, Vkamali@basu.ac.ir

1 Introduction

Big Bang model have many long-standing problems (horizon, flatness,...). These problems are solved in the framework of the inflationary universe models [1]. Scalar field as a source of inflation provides the causal interpretation of the origin of the distribution of large scale structure and observed anisotropy of cosmological microwave background (CMB) [2]. In standard models for inflationary universe, the inflation period is divided into two regimes, slow-roll and reheating epochs. In slow-roll period kinetic energy remains small compared to the potential terms. In this period, all interactions between scalar fields (inflaton) and other fields are neglected and the universe inflates. Subsequently, in reheating period, the kinetic energy is comparable to the potential energy and inflaton starts an oscillation around the minimum of the potential losing their energy to other fields present in the theory. So, the reheating is the end period of inflation.

In warm inflationary models radiation production occurs during inflationary period and reheating is avoided [3]. Thermal fluctuations may be obtained during warm inflation. These fluctuations could play a dominant role to produce initial fluctuations which are necessary for Large-Scale Structure (LSS) formation. So the density fluctuation arises from thermal rather than quantum fluctuation [4]. Warm inflationary period ends when the universe stops inflating. After this period the universe enters in radiation phase smoothly [3]. Finally, remaining inflatons or dominant radiation fields created the matter components of the universe.

Friedmann-Robertson-Walker (FRW) cosmological models in the context of string/M-theory have related to brane-antibrane configurations [5]. Tachyon fields associated with unstable D-branes may be responsible for inflation in early time [6]. The tachyon inflation is a k-inflation model [7] for scalar field ϕ with a positive potential $V(\phi)$. Tachyon potentials have two special properties, firstly a maximum of these potential is obtained where $\phi \rightarrow 0$ and second property is the minimum of these potentials is obtained where $\phi \rightarrow \infty$. If the tachyon field start to roll down the potential, then universe dominated by a new form of matter will smoothly evolve from inflationary universe to an era which is dominated by a non-relativistic fluid [8]. So, we could explain the phase of acceleration expansion (inflation) in term of tachyon field.

The warm tachyon inflationary model have been studied in Ref.[9]. In the present work we will study warm-tachyon inspired inflation in the context of the effective theory of loop quantum gravity (LQG). Techniques of LQG which is a resulting non-perturbative background independent approach to

quantizing gravity [10], could be applied in homogeneous and isotropic space-time which is known as loop quantum cosmology (LQC). Canonical quantization gravity in term of Ashtekar-Barbero connection variables is studied in LQG. In LQG the phase space of classical general relativity may be spanned by conjugate variables A_a^i (connection) and E_i^a (triad) on a 3-manifold \mathcal{M} which encode curvature and spatial geometry respectively (labels a and i denote internal indices of $SU(2)$ and space index respectively). Due to the isotropic and homogeneous symmetries, in LQC model the phase space is simplified. The phase space of this model is spanned by a single connection \mathbf{c} and a single triad \mathbf{p} . The Poisson bracket for LQC variable is given by

$$\{\mathbf{c}, \mathbf{p}\} = \frac{8\pi\gamma}{3m_p^2}$$

where γ is the dimensionless Barbere-Immirzi parameter. For spatially flat (FRW) universe the LQC variables \mathbf{c} and \mathbf{p} have these relations with the metric components

$$\mathbf{c} = \gamma \dot{a} \quad \mathbf{p} = a^2$$

Classical Hamiltonian constraint in term of connection and triad variables is given by

$$\mathcal{H}_{cl} = -\frac{3\sqrt{\mathbf{p}}}{\gamma^2} + \mathbf{c}^2 + \mathcal{H}$$

where \mathcal{H}_m is the matter Hamiltonian. In Hamiltonian formalism, the dynamical equations (modified Friedmann equation) may be determined by the above Hamiltonian constraint. The effective classical Hamiltonian constraint in terms of kinematical length of the edge of square loop $\bar{\mu}$ is given by [11]

$$\mathcal{H} = -\frac{3}{\gamma\bar{\mu}^2} a \sin^2(\mu\mathbf{c}) + \mathcal{H}_m$$

Using the Hamilton equation of motion

$$\dot{\mathbf{p}} = \{\mathbf{p}, \mathcal{H}_{eff}\} = -\frac{\gamma}{3} \frac{\partial \mathcal{H}_{eff}}{\partial \mathbf{c}}$$

and the vanishing of the Hamiltonian constraint ($\mathcal{H}_{eff} \approx 0$) [11], these two important relations are obtained

$$\begin{aligned} \dot{a} &= \frac{1}{\gamma\bar{\mu}} \sin(\bar{\mu}\mathbf{c}) \cos(\bar{\mu}\mathbf{c}) \\ \sin^2(\bar{\mu}\mathbf{c}) &= \frac{8\pi}{3m_p^2 a} \mathcal{H}_m \end{aligned}$$

Therefore, from above equation the modified Friedmann equation becomes

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi}{3m_p^2}\rho\left[1 - \frac{\rho}{\rho_c}\right] \quad \rho_c = \frac{4\sqrt{3}}{\gamma^3} \quad (1)$$

In this paper we will study warm-tachyon inflationary model in the context of LQC by using the above modified Friedmann equation. The paper organized as: In the next section we will describe warm-tachyon inflationary universe model in the framework of LQC. In section (3) we consider the perturbations for our model and obtain scalar and tensor perturbation spectrums. In section (4) we study our model using the exponential potential in high dissipative regime. Finally in section (5) we close by some concluding remarks.

2 The model

In the present work we will study warm-tachyon inspired inflation in the context of effective field theory of LQC where the modified Friedmann equation has the following form

$$H^2 = \frac{1}{3}[\rho_\phi + \rho_\gamma]\left[1 - \frac{\rho_\phi + \rho_\gamma}{\rho_c}\right] \quad (2)$$

where $H = \frac{\dot{a}}{a}$ is the Hubble factor, a is the scale factor and we choose $c = \hbar = 8\pi G = \frac{8\pi}{m_p^2} = 1$ (m_p is Planck mass.). Energy-momentum tensor of tachyonic inflation model in a spatially flat Friedmann Robertson Walker (FRW) is recognized by $T_\mu^\nu = \text{diag}(-\rho_\phi, P_\phi, P_\phi, P_\phi)$ where the pressure and energy density of tachyon field are defined by [8]

$$P_\phi = -V(\phi)\sqrt{1 - \dot{\phi}^2} \quad (3)$$

and

$$\rho_\phi = \frac{V(\phi)}{\sqrt{1 - \dot{\phi}^2}} \quad (4)$$

respectively, where $V(\phi)$ is the effective scalar potential associated with tachyon field ϕ . Important characteristics of this potential are $\frac{dV}{d\phi} < 0$ and $V(\phi \rightarrow 0) \rightarrow V_{max}$ [12]. The dynamic of warm tachyon inflation in spatially

flat FRW model in the context of effective theory LQC is described by these equations.

$$H^2 = \frac{1}{3} \left[\frac{V(\phi)}{\sqrt{1-\dot{\phi}^2}} + \rho_\gamma \right] \left[1 - \frac{1}{\rho_c} \left(\frac{V(\phi)}{\sqrt{1-\dot{\phi}^2}} \right) \right] \quad (5)$$

$$\dot{\rho}_\phi + 3H(P_\phi + \rho_\phi) = -\Gamma\dot{\phi}^2 \Rightarrow \frac{\ddot{\phi}}{1-\dot{\phi}^2} + 3H\dot{\phi} + \frac{V'}{V} = -\frac{\Gamma}{V}\sqrt{1-\dot{\phi}^2}\dot{\phi} \quad (6)$$

and

$$\dot{\rho}_\gamma + 4H\rho_\gamma = \Gamma\dot{\phi}^2 \quad (7)$$

where ρ_γ is the energy density of the radiation and Γ is the dissipative coefficient with the dimension m_p^5 . In the above equations dots "." mean derivative with respect to cosmic time and prime denotes derivative with respect to scalar field ϕ . During inflation epoch the energy density (4) is the order of potential $\rho_\phi \sim V$ and dominates over the radiation energy $\rho_\phi > \rho_\gamma$. Using slow-roll approximation when $\dot{\phi} \ll 1$ and $\ddot{\phi} \ll (3H + \frac{\Gamma}{V})$ [3] and when inflation radiation production is quasi-stable ($\dot{\rho}_\gamma \ll 4H\Gamma$, $\dot{\rho}_\gamma \ll \Gamma\dot{\phi}^2$) the dynamic equations (5) and (6) are reduced to

$$H^2 = \frac{1}{3}V\left(1 - \frac{V}{\rho_c}\right) \quad (8)$$

$$3H(1+r)\dot{\phi} = -\frac{V'}{V} \quad (9)$$

where $r = \frac{\Gamma}{3HV}$. From above equations and Eq.(7), ρ_γ could be written as

$$\rho_\gamma = \frac{\Gamma\dot{\phi}^2}{4H} = \frac{r}{4(1+r)^2\left(1 - \frac{V}{\rho_c}\right)} \left(\frac{V'}{V}\right)^2 = \sigma T_r^4 \quad (10)$$

where T_r is the temperature of thermal bath and σ is Stefan-Boltzmann constant. We introduce the slow-roll parameters for our model as

$$\epsilon = -\frac{\dot{H}}{H^2} \simeq \frac{V'^2}{2(1+r)V^3} \frac{1 - \frac{2V}{\rho_c}}{1 - \frac{V}{\rho_c}} \quad (11)$$

and

$$\begin{aligned} \eta = -\frac{\ddot{H}}{H\dot{H}} &\simeq \frac{2V'}{V^2(1+r)\left[1 - \frac{V}{\rho_c}\right]} \\ &\times \left[\frac{V''}{V'} - \frac{V'}{V} - \frac{r'}{2(1+r)} - \frac{V'}{\rho_c - 2V} + \frac{V'}{2\rho_c - 2V} \right] \end{aligned} \quad (12)$$

A relation between two energy densities ρ_ϕ and ρ_γ is obtained from Eqs. (10) and (11)

$$\rho_\gamma = \frac{r}{2(1+r)} \frac{[1 - \frac{\rho_\phi}{\rho_c}]}{[1 - \frac{2\rho_\phi}{\rho_c}]} \rho_\phi \epsilon \simeq \frac{r}{2(1+r)} \frac{[1 - \frac{V}{\rho_c}]}{[1 - \frac{2V}{\rho_c}]} V \epsilon \quad (13)$$

The condition of inflation epoch $\ddot{a} > 1$ could be obtained by inequality $\epsilon < 1$. Therefore from above equation, warm-tachyon inflation in the context of effective theory LQC could take place when

$$\frac{2(1+r)}{r} \rho_\gamma < \frac{1 - \frac{\rho_\phi}{\rho_c}}{1 - \frac{2\rho_\phi}{\rho_c}} \quad (14)$$

Inflation period ends when $\epsilon \simeq 1$ which implies

$$[\frac{V'_f}{V_f}]^2 \frac{1 - \frac{2V}{\rho_c}}{1 - \frac{V}{\rho_c}} \frac{1}{V_f} \simeq 2(1+r_f) \quad (15)$$

where the subscript f denotes the end of inflation. The number of e-folds is given by

$$N = \int_{\phi_*}^{\phi_f} H dt = \int_{\phi_*}^{\phi_f} \frac{H}{\dot{\phi}} d\phi = - \int_{\phi_*}^{\phi_f} \frac{V^2}{V'} (1+r) [1 - \frac{V}{\rho_c}] d\phi \quad (16)$$

where the subscript $*$ denotes the epoch when the cosmological scale exits the horizon.

3 Perturbation

In quantum cosmology the interesting primary quantities are the curvature and tensor perturbation spectrums which may be extracted from two-point function of two quantum fields in the same time. In this section we will study the cosmological perturbations for our model in high dissipative regime ($r \gg 1$) that lead to the perturbation spectrums [13]. Sclar perturbations in the longitudinal gauge, may be described by the perturbed FRW metric

$$ds^2 = (1 + 2\Phi)dt^2 - a^2(t)(1 - 2\Psi)\delta_{ij}dx^i dx^j \quad (17)$$

where Φ and Ψ are gauge-invariant metric perturbation variables [13]. The equation of motion is given by

$$\begin{aligned} \frac{\ddot{\delta\phi}}{1-\dot{\phi}^2} + [3H + \frac{\Gamma}{V}]\dot{\delta\phi} + [-a^{-2}\nabla^2 + (\ln V)'' + \dot{\phi}(\frac{\Gamma}{V})']\delta\phi \\ - [\frac{1}{1-\dot{\phi}^2} + 3]\dot{\phi}\dot{\Phi} - [\dot{\phi}\frac{\Gamma}{V} - 2(\ln V)']\Phi = 0 \end{aligned} \quad (18)$$

We expand the small change of field $\delta\phi$ into Fourier components as

$$\delta\phi(x) = \int \frac{d^3k}{(2\pi)^3} [e^{ikx}\delta\phi(k,t)\vec{a}_k + e^{-ikx}\delta\phi(k,t)\vec{a}_k^\dagger] \quad (19)$$

where a_k and a_k^\dagger denote the annihilation and creation operators respectively. These operators obey the simple commutation relations

$$[a_k, a_{k'}^\dagger] = (2\pi)^3 \delta^3(k - k') \quad [a_k, a_{k'}] = 0 = [a_k^\dagger, a_{k'}^\dagger] \quad (20)$$

All perturbed quantities have a spatial sector of the form $e^{i\mathbf{k}\mathbf{x}}$, where k is the wave number. Perturbed Einstein field equations in momentum space have only the temporal parts

$$\Phi = \Psi$$

$$\dot{\Phi} + H\Phi = \frac{1}{2}[-\frac{4\rho_\gamma av}{3k} + \frac{V\dot{\phi}}{\sqrt{1-\dot{\phi}^2}}\delta\phi][1 - \frac{2}{\rho_c}[\rho_\gamma + \frac{V}{\sqrt{1-\dot{\phi}^2}}]] \quad (21)$$

$$\begin{aligned} \frac{\ddot{\delta\phi}}{1-\dot{\phi}^2} + [3H + \frac{\Gamma}{V}]\dot{\delta\phi} + [\frac{k^2}{a^2} + (\ln V)'' + \dot{\phi}(\frac{\Gamma}{V})']\delta\phi \\ - [\frac{1}{1-\dot{\phi}^2} + 3]\dot{\phi}\dot{\Phi} - [\dot{\phi}\frac{\Gamma}{V} - 2(\ln V)']\Phi = 0 \end{aligned} \quad (22)$$

$$(\delta\dot{\rho}_\gamma) + 4H\delta\rho_\gamma + \frac{4}{3}ka\rho_\gamma v - 4\rho_\gamma\dot{\Phi} - \dot{\phi}^2\Gamma'\delta\phi - \Gamma\dot{\phi}^2[2(\delta\dot{\phi}) - 3\dot{\phi}\Phi] = 0 \quad (23)$$

and

$$\dot{v} + 4Hv + \frac{k}{a}[\Phi + \frac{\delta\rho_\gamma}{4\rho_\gamma} + \frac{3\Gamma\dot{\phi}}{4\rho_\gamma}\delta\phi] \quad (24)$$

The above equations are obtained for Fourier components $e^{i\mathbf{k}\mathbf{x}}$, where the subscript k is omitted. v in the above set of equations is given by the decomposition of the velocity field ($\delta u_j = -\frac{iak_J}{k}ve^{i\mathbf{k}\mathbf{x}}, j = 1, 2, 3$) [13].

Warm inflation models could be considered as a hybrid-like inflationary model where inflaton field interacts with radiation field [14], [15]. Entropy perturbation relates to dissipation term [16]. During slow-roll inflationary phase, for non-decreasing adiabatic modes on large scale limit $k \ll aH$, we assume that the perturbed quantities do not vary strongly. So we constrain above equation as: $H\Phi \gg \dot{\Phi}$, $(\delta\dot{\phi}) \ll (\Gamma + 3H)(\delta\phi)$, $(\delta\dot{\rho}_\gamma) \ll \delta\rho_\gamma$ and $\dot{v} \ll 4Hv$. In the slow-roll limit, and by using the above limitations, the set of perturbed equations reduce to

$$\Phi \simeq \frac{1}{H^2} \left[-\frac{4\rho_\gamma a v}{3k} + V \dot{\phi} \delta\phi \right] \left[1 - \frac{2V}{\rho_c} \right] \quad (25)$$

$$\left[3H + \frac{\Gamma}{V} \right] \delta\dot{\phi} + [(\ln V)'' + \dot{\phi} \left(\frac{\Gamma}{V} \right)'] \delta\phi \simeq \left[\dot{\phi} \frac{\Gamma}{V} - 2(\ln V)' \right] \Phi \quad (26)$$

$$\frac{\delta\rho_\gamma}{\rho_\gamma} \simeq \frac{\Gamma'}{\Gamma} \delta\phi - 3\Phi \quad (27)$$

and

$$v \simeq -\frac{k}{4aH} \left(\Phi + \frac{\delta\rho_\gamma}{4\rho_\gamma} + \frac{3\Gamma\dot{\phi}}{4\rho_\gamma} \delta\phi \right) \quad (28)$$

Using Eqs.(25), (27) and (28) we determine the perturbation variable Φ :

$$\Phi = \frac{V\dot{\phi}}{2H} \left[1 + \frac{\Gamma}{4HV} + \frac{\Gamma'\dot{\phi}}{48H^2V} \right] \left(1 - \frac{2V}{\rho_c} \right) \delta\phi \quad (29)$$

We can solve the above equations by taking inflaton ϕ as the independent variable in place of cosmic time t . Using Eq.(9) we find

$$\left(3H + \frac{\Gamma}{V} \right) \frac{d}{dt} = \left(3H + \frac{\Gamma}{V} \right) \dot{\phi} \frac{d}{d\phi} = -\frac{V'}{V} \frac{d}{d\phi} \quad (30)$$

From above equation, Eq.(26) and Eq.(29), the expression $\frac{(\delta\phi)'}{\delta\phi}$ is obtained

$$\begin{aligned} \frac{(\delta\phi)'}{\delta\phi} &= \frac{1}{(\ln V)'} \left[(\ln V)'' + \dot{\phi} \left(\frac{\Gamma}{V} \right)' + \frac{1}{2} \left(-\dot{\phi} \frac{\Gamma}{V} + 2(\ln V)' \right) \right] \\ &\times \left(\frac{V\dot{\phi}}{H} \right) \left[1 + \frac{\Gamma}{4HV} + \frac{\Gamma'\dot{\phi}}{48H^2V} \right] \left(1 - \frac{2V}{\rho_c} \right) \end{aligned} \quad (31)$$

We will return to the above relation soon. Following Refs.[9], [16], [17], we introduce auxiliary function χ as

$$\chi = \frac{\delta\phi}{(\ln V)'} \exp\left[\int \frac{1}{3H + \frac{\Gamma}{V}} \left(\frac{\Gamma}{V}\right)' d\phi\right] \quad (32)$$

From above definition we have

$$\frac{\chi'}{\chi} = \frac{(\delta\phi)'}{\delta\phi} - \frac{(\ln V)''}{(\ln V)'} + \frac{(\frac{\Gamma}{V})'}{3H + \frac{\Gamma}{V}} \quad (33)$$

Using above equation and Eq.(50) we find

$$\frac{\chi'}{\chi} = \frac{1}{2} \left(-\frac{\dot{\phi}}{(\ln V)'} \frac{\Gamma}{V} + 2 \right) \left(\frac{V\dot{\phi}}{H} \right) \left[1 + \frac{\Gamma}{4HV} + \frac{\Gamma'\dot{\phi}}{48H^2V} \right] \left(1 - \frac{2V}{\rho_c} \right) \quad (34)$$

We could rewrite this equation, using Eqs. (8) and (9)

$$\frac{\chi'}{\chi} = -\frac{9}{8} \frac{2H + \frac{\Gamma}{V}}{(3H + \frac{\Gamma}{V})^2} \left(\Gamma + 4HV - \frac{\Gamma'(\ln V)'}{12H(3H + \frac{\Gamma}{V})} \right) \frac{(\ln V)'}{V} \frac{[1 - \frac{2V}{\rho_c}]}{1 - \frac{V}{\rho_c}} \quad (35)$$

A solution for the above equation is

$$\begin{aligned} \chi(\phi) = & C \exp\left(-\int \left\{ -\frac{9}{8} \frac{2H + \frac{\Gamma}{V}}{(3H + \frac{\Gamma}{V})^2} \right. \right. \\ & \times \left. \left(\Gamma + 4HV - \frac{\Gamma'(\ln V)'}{12H(3H + \frac{\Gamma}{V})} \right) \frac{(\ln V)'}{V} \frac{[1 - \frac{2V}{\rho_c}]}{1 - \frac{V}{\rho_c}} \right\} d\phi \end{aligned} \quad (36)$$

where C is integration constant. From above equation and Eq.(52) the change of variable $\delta\phi$ is determined

$$\delta\phi = C(\ln V)' \exp(\Im(\phi)) \quad (37)$$

where

$$\begin{aligned} \Im(\phi) = & -\int \left[\frac{(\frac{\Gamma}{V})'}{3H + \frac{\Gamma}{V}} + \left(\frac{9}{8} \frac{2H + \frac{\Gamma}{V}}{(3H + \frac{\Gamma}{V})^2} \right. \right. \\ & \times \left. \left(\Gamma + 4HV - \frac{\Gamma'(\ln V)'}{12H(3H + \frac{\Gamma}{V})} \right) \frac{(\ln V)'}{V} \frac{[1 - \frac{2V}{\rho_c}]}{1 - \frac{V}{\rho_c}} \right) \Big] d\phi \end{aligned} \quad (38)$$

In the above calculations we have used the perturbation method in the warm inflation models [9], [17], [16], where the small change of variable $\delta\phi$ could

be generated by thermal fluctuations instead of quantum fluctuations [21], and the integration constant C may be driven by boundary conditions for field perturbation. Perturbed matter fields of our model are radiation $\delta\rho_r$, inflaton $\delta\phi$ and velocity $k^{-1}(P + \rho)v_{,i}$. We can explain the cosmological perturbations in terms of gauge-invariant variables. These variables are important for development of perturbation after the end of inflation period. The curvature perturbation \mathfrak{R} and entropy perturbation e are defined by [18]

$$\begin{aligned}\mathfrak{R} &= \Phi - k^{-1}aHv \\ e &= \delta P - c_s^2\delta\rho\end{aligned}\tag{39}$$

where $c_s^2 = \frac{\dot{P}}{\dot{\rho}}$. The boundary condition of warm inflation models are found in very large scale limits i.e., $k \ll aH$ where the curvature perturbation $\mathfrak{R} \sim \text{const}$ and the entropy perturbation vanishes [19].

Finally the density perturbation is presented by [20]

$$\delta_H = \frac{16\pi}{5} \frac{\exp(-\mathfrak{I}(\phi))}{(\ln V)'} \delta\phi = \frac{16\pi}{15} \frac{\exp(-\mathfrak{I}(\phi))}{Hr\dot{\phi}} \delta\phi\tag{40}$$

In warm inflation model the fluctuations of the scalar field in high dissipative regime ($r \gg 1$) may be generated by thermal fluctuation instead of quantum fluctuations [21] as

$$(\delta\phi)^2 \simeq \frac{k_F T_r}{2\pi^2}\tag{41}$$

where in this limit freeze-out wave number $k_F = \sqrt{\frac{\Gamma H}{V}} = H\sqrt{3r} \geq H$ corresponds to the freeze-out scale at the point when, dissipation damps out to thermally excited fluctuations ($\frac{V''}{V} < \frac{\Gamma H}{V}$) [21]. With the help of the above equation and Eq.(40) high dissipative regime ($r \gg 1$) we find

$$\delta_H^2 = \frac{128\sqrt{3}}{75} \frac{\exp(-2\tilde{\mathfrak{I}}(\phi))}{\sqrt{r}\tilde{\epsilon}} \frac{T_r}{H}\tag{42}$$

where

$$\tilde{\mathfrak{I}}(\phi) = - \int \left[\frac{1}{3Hr} \left(\frac{\Gamma}{V} \right)' + \frac{9}{4} \left(1 - \frac{(\ln \Gamma)'(\ln V)'}{36rH^2} \right) (\ln V)' \frac{[1 - \frac{2V}{\rho_c}]}{1 - \frac{V}{\rho_c}} \right] d\phi\tag{43}$$

and

$$\tilde{\epsilon} = \frac{V'^2}{2rV^3} \frac{1 - \frac{2V}{\rho_c}}{1 - \frac{V}{\rho_c}}\tag{44}$$

An important perturbation parameter is scalar index n_s which in high dissipative regime is given by

$$n_s = 1 + \frac{d \ln \delta_H^2}{d \ln k} \approx 1 - \frac{3}{4}\tilde{\epsilon} + \frac{3}{4}\tilde{\eta} + \left(\frac{\tilde{\epsilon}}{3r}\right)^{\frac{1}{2}}(2\tilde{\mathfrak{S}}'(\phi) + \frac{r'}{2r}) \quad (45)$$

where

$$\tilde{\eta} = \frac{2V'}{V^2 r [1 - \frac{V}{\rho_c}]} \left[\frac{V''}{V'} - \frac{V'}{V} - \frac{r'}{2r} - \frac{V'}{\rho_c - 2V} + \frac{V'}{2\rho_c - 2V} \right] \quad (46)$$

In Eq.(45) we have used a relation between small change of the number of e-folds and interval in wave number ($dN = -d \ln k$). During inflation epoch, there are two independent components of gravitational waves ($h_{\times+}$) with action of massless scalar field that are produced by the generation of tensor perturbations. The amplitude of tensor perturbation is given by

$$A_g^2 = \frac{1}{4\pi} \left(\frac{H}{2\pi}\right)^2 \coth\left[\frac{k}{2T}\right] \quad (47)$$

where, the temperature T in extra factor $\coth[\frac{k}{2T}]$, denotes the temperature of the thermal background of gravitational wave [22]. Spectral index n_g may be found as

$$n_g = \frac{d}{d \ln k} (\ln[\frac{A_g^2}{\coth(\frac{k}{2T})}]) \simeq -2\tilde{\epsilon} \quad (48)$$

where $A_g \propto k^{n_g} \coth[\frac{k}{2T}]$ [22]. Using Eqs. (42) and (47) we write the tensor-scalar ratio in high dissipative regime

$$R(k) = \frac{A_g^2}{P_R} \Big|_{k=k_0} \quad (49)$$

where k_0 is referred to pivot point [22] and $P_R = \frac{25}{4}\delta_H^2$. An upper bound for this parameter is obtained by using WMAP9 and BICEP2 observational data, $R < 0.36$ [2].

4 Exponential potential

In this section we consider our model with the tachyonic effective potential

$$V(\phi) = V_0 \exp(-\alpha\phi) \quad (50)$$

where parameter $\alpha > 0$ (with unit m_p) is related to mass of tachyon field [23]. The exponential form of potential have characteristics of tachyon field ($\frac{dV}{d\phi} < 0$ and $V(\phi \rightarrow 0) \rightarrow V_{max}$). We develop our model in high dissipative regime i.e. $r \gg 1$ for a constant dissipation coefficient Γ . By using Eq.(9) and potential (50), the scalar field in terms cosmic time is found

$$\phi(t) = \frac{1}{\alpha} \ln[\exp(\alpha\phi_i) + \frac{\alpha^2 V_0}{\Gamma} t] \quad (51)$$

where $\phi(t = t_i = 0) = \phi_i$. Using above equation, Eqs.(8) and (50) we find the potential and Hubble parameter as

$$V(t) = \frac{V_0}{\exp(\alpha\phi_i) + \frac{\alpha^2 V_0}{\Gamma} t} \quad (52)$$

$$H^2 = \frac{1}{3} \frac{V_0}{\exp(\alpha\phi_i) + \frac{\alpha^2 V_0}{\Gamma} t} \left[1 - \frac{V_0/\rho_c}{\exp(\alpha\phi_i) + \frac{\alpha^2 V_0}{\Gamma} t} \right]$$

Dissipative parameter $r = \frac{\Gamma}{3HV}$ in this case becomes

$$r = \frac{\Gamma_0}{\sqrt{3}} \frac{(\exp(\alpha\phi_i) + \alpha^2 \frac{V_0}{\Gamma_0} t)^{\frac{3}{2}}}{V_0^{\frac{3}{2}} (1 - \frac{\exp(\alpha\phi_i) + \alpha^2 \frac{V_0}{\Gamma_0} t}{V_0 \rho_c})^{\frac{1}{2}}} \quad (53)$$

Using Eq.(10) we find a relation between the energy densities of radiation and inflaton fields.

$$\rho_\gamma = \frac{3\rho_\phi^{\frac{3}{2}}}{4\sqrt{3}\Gamma_0(1 - \frac{\rho_\phi}{\rho_c})^{\frac{1}{2}}} \quad (54)$$

Power-spectrum in this case becomes (from Eq.(23))

$$P_R = \frac{3^{\frac{5}{4}} \exp(-2\tilde{\mathfrak{S}}(\phi)) \Gamma_0^{\frac{5}{4}} T_r}{2\pi^2 \alpha^2} V^{\frac{5}{4}} (1 - \nu)^{\frac{5}{4}} \quad (55)$$

where $\nu = \frac{V}{\rho_c}$ describes the quantum geometry effects in LQC and $\tilde{\mathfrak{S}}(\phi) = -\frac{9}{4} \ln V$. From Eq.(30) we find the tensor-scalar ratio as

$$R = \frac{4 \exp(2\tilde{\mathfrak{S}}(\phi)) \times 3^{\frac{1}{4}} \alpha^2}{3T_r \Gamma_0^{\frac{5}{2}}} \frac{V^{\frac{7}{4}}}{(1 - \nu)^{\frac{1}{4}}} \coth\left[\frac{k}{2T}\right] \quad (56)$$

From observational data, we know $P_R = 2.28 \times 10^{-9}$ and $R = 0.21 < 0.36$ [2]. From above equations and WMAP7 data we find an upper bound for the potential

$$V_* < 3.4 \times 10^{-4} \quad (57)$$

We have obtained above equation in $\nu < 1$ limit. By using BICEP2 data, we have found a new maximum of V_* (See for example [24]).

5 Conclusion

Tachyon inflation model with overlasting form of potential $V(\phi) = V_0 \exp(-\alpha\phi)$ which agrees with tachyon potential properties have been studied. The main problem of inflation theory is how to attach the universe to the end of the inflation period. One of the solutions of this problem is the study of inflation in the context of warm inflation [3]. In this model radiation is produced during inflation period where its energy density is kept nearly constant. This is phenomenologically fulfilled by introducing the dissipation term Γ . The study of warm inflation model as a mechanism that gives an end for tachyon inflation are motivated us to consider the warm tachyon inflation model. In this article we have considered warm-tachyon inflationary universe model in the framework of effective field theory LQC. In slow-roll approximation the explicit expressions for the tensor-scalar ratio R , scalar spectrum P_R and index n_s have been presented. We have developed our specific model by exponential potential. In this case we have presented perturbation parameters and constrained this parameters by observational data. We also have constrained the exponential potential by using these data.

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